Contents

CHAPTER 2

Exercise 2.3

1. (a) $\{x \mid x > 34\}$ (b) $\{x \mid 8 < x < 65\}$

2. True statements: (a), (d), (f), (g), and (h)

3. (a) {2,4,6,7} (b) {2,4,6} (c) {2,6}
   (d) {2} (e) {2} (f) {2,4,6}

4. All are valid.

5. First part: $A \cup (B \cap C) = \{4, 5, 6\} \cup \{3, 6\} = \{3, 4, 5, 6\}$; and $(A \cup B) \cap (A \cup C) = \{3, 4, 5, 6, 7\} \cap \{2, 3, 4, 5, 6\} = \{3, 4, 5, 6\}$ too.
   Second part: $A \cap (B \cup C) = \{4, 5, 6\} \cap \{2, 3, 4, 6, 7\} = \{4, 6\}$; and $(A \cap B) \cup (A \cap C) = \{4, 6\} \cup \{6\} = \{4, 6\}$ too.

6. N/A

7. $\emptyset$, $\{5\}$, $\{6\}$, $\{7\}$, $\{5, 6\}$, $\{5, 7\}$, $\{6, 7\}$, $\{5, 6, 7\}$

8. There are $2^4 = 16$ subsets: $\emptyset$, {a}, {b}, {c}, {d}, {a,b}, {a,c}, {a,d}, {b,c}, {b,d}, {c,d}, {a,b,c}, {a,b,d}, {a,c,d}, {b,c,d}, and {a,b,c,d}.

9. The complement of U is $\tilde{U} = \{x \mid x \notin U\}$. Here the notation of "not in U" is expressed via the $\notin$ symbol which relates an *element* (x) to a *set* (U). In contrast, when we say "$\emptyset$ is a subset of U," the notion of "in U" is expressed via the $\subset$ symbol which relates a subset($\emptyset$) to a set (U). Hence, we have two different contexts, and there exists no paradox at all.

Exercise 2.4

1. (a) {(3,a), (3,b), (6,a), (6,b) (9,a), (9,b)}
   (b) {(a,m), (a,n), (b,m), (b,n)}
   (c) { (m,3), (m,6), (m,9), (n,3), (n,6), (n,9)}

2. {(3,a,m), (3,a,n), (3,b,m), (3,b,n), (6,a,m), (6,a,n), (6,b,m), (6,b,n), (9,a,m), (9,a,n), (9,b,m), (9,b,n),}

3. No. When $S_1 = S_2$.

4. Only (d) represents a function.

5. Range $= \{y \mid 8 \leq y \leq 32\}$

6. The range is the set of all nonpositive numbers.

7. (a) No. (b) Yes.

8. For each level of output, we should discard all the inefficient cost figures, and take the lowest cost figure as the total cost for that output level. This would establish the uniqueness as required by the definition of a function.

Exercise 2.5

1. N/a

2. Eqs. (a) and (b) differ in the sign of the coefficient of x; a positive (negative) sign means an upward (downward) slope.
   Eqs. (a) and (c) differ in the constant terms; a larger constant means a higher vertical intercept.

3. A negative coefficient (say, -1) for the x^2 term is associated with a hill. as the value of x is steadily increased or reduced, the $-x^2$ term will exert a more dominant influence in determining the value of y. Being negative, this term serves to pull down the y values at the two extreme ends of the curve.

4. If negative values can occur there will appear in quadrant III a curve which is the mirror image of the one in quadrant I.

5. (a) x^{19} (b) x^{a+b+c} (c) $(xyz)^3$

6. (a) x^6 (b) $x^{1/6}$

7. By Rules VI and V, we can successively write $x^{m/n} = (x^m)^{1/n} = \sqrt[n]{x^m}$; by the same two rules, we also have $x^{m/n} = (x^{1/n})^m = (\sqrt[n]{x})^m$

8. Rule VI:

$$(x^m)^n = \underbrace{x^m \times x^m \times ... \times x^m}_{\text{n terms}} = \underbrace{x \times x \times ... \times x}_{mn \text{ terms}} = x^{mn}$$

Rule VII:

$$\begin{aligned} x^m \times y^m &= \underbrace{x \times x \times ... \times x}_{m \text{ terms}} \times \underbrace{y \times y \ldots \times y}_{m \text{ terms}} \\ &= \underbrace{(xy) \times (xy) \times \ldots \times (xy)}_{m \text{ terms}} = (xy)^m \end{aligned}$$

CHAPTER 3

Exercise 3.2

1. (a) By substitution, we get $21 - 3P = -4 + 8P$ or $11P = 25$. Thus $P^* = 2\frac{3}{11}$. Substituting P^* into the second equation or the third equation, we find $Q^* = 14\frac{2}{11}$.

   (b) With $a = 21$, $b = 3, c = 4, d = 8$, the formula yields

$$P^* = \tfrac{25}{11} = 2\tfrac{3}{11} \quad Q^* = \tfrac{156}{11} = 14\tfrac{2}{11}$$

2.

   (a)

$$P^* = \tfrac{61}{9} = 6\tfrac{7}{9} \quad Q^* = \tfrac{276}{9} = 30\tfrac{2}{3}$$

   (b)

$$P^* = \tfrac{36}{7} = 5\tfrac{1}{7} \quad Q^* = \tfrac{138}{7} = 19\tfrac{5}{7}$$

3. N/A

4. If $b+d = 0$ then P^* and Q^* in (3.4) and (3.5) would involve division by zero, which is undefined.

5. If $b + d = 0$ then $d = -b$ and the demand and supply curves would have the same slope (though different vertical intercepts). The two curves would be parallel, with no equilibrium intersection point in Fig. 3.1

Exercise 3.3

1. (a) $x_1^* = 5;$ $x_2^* = 3$ (b) $x_1^* = 4;$ $x_2^* = -2$

2. (a) $x_1^* = 5;$ $x_2^* = 3$ (b) $x_1^* = 4;$ $x_2^* = -2$

3.

   (a) $(x-6)(x+1)(x-3) = 0$, or $x^3 - 8x^2 + 9x + 18 = 0$

   (b) $(x-1)(x-2)(x-3)(x-5) = 0$, or $x^4 - 11x^3 + 41x^2 - 61x + 30 = 0$

4. By Theorem III, we find:

   (a) Yes. (b) No. (c) Yes.

5.

(a) By Theorem I, any integer root must be a divisor of 6; thus there are six candidates: ± 1, ± 2, and ± 3. Among these, -1, $\frac{1}{2}$, and $-\frac{1}{4}$

(b) By Theorem II, any rational root r/s must have r being a divisor of -1 and s being a divisor of 8. The r set is $\{1, -1\}$, and the s set is $\{1, -1, 2, -2, 4, -4, 8, -8\}$; these give us eight root candidates: $\pm 1, \pm\frac{1}{2}, \pm\frac{1}{4}$, and $\pm\frac{1}{8}$. Among these, -1, 2, and 3 satisfy the equation, and they constitute the three roots.

(c) To get rid of the fractional coefficients, we multiply every term by 8. The resulting equation is the same as the one in (b) above.

(d) To get rid of the fractional coefficients, we multiply every term by 4 to obtain

$$4x^4 - 24x^3 + 31x^2 - 6x - 8 = 0$$

By Theorem II, any rational root r/s must have r being a divisor of -8 and s being a divisor of 4. The r set is $\{\pm 1, \pm 2, \pm 4, \pm 8\}$, and the s set is $\{\pm 1, \pm 2, \pm 4\}$; these give us the root candidates $\pm 1, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm 2, \pm 4, \pm 8$. Among these, $\frac{1}{2}$, $-\frac{1}{2}$, 2, and 4 constitute the four roots.

6.

(a) The model reduces to $P^2 + 6P - 7 = 0$. By the quadratic formula, we have $P_1^* = 1$ and $P_2^* = -7$, but only the first root is acceptable. Substituting that root into the second or the third equation, we find $Q^* = 2$.

(b) The model reduces to $2P^2 - 10 = 0$ or $P^2 = 5$ with the two roots $P_1^* = \sqrt{5}$ and $P_2^* = -\sqrt{5}$. Only the first root is admissible, and it yields $Q^* = 3$.

7. Equation (3.7) is the equilibrium stated in the form of "the excess supply be zero."

Exercise 3.4

1. N/A

2.

$$P_1^* = \frac{(a_2-b_2)(\alpha_0-\beta_0)-(a_0-b_0)(\alpha_2-\beta_2)}{(a_1-b_1)(\alpha_2-\beta_2)-(a_2-b_2)(\alpha_1-\beta_1)}$$

$$P_2^* = \frac{(a_0-b_0)(\alpha_1-\beta_1)-(a_1-b_1)(\alpha_0-\beta_0)}{(a_1-b_1)(\alpha_2-\beta_2)-(a_2-b_2)(\alpha_1-\beta_1)}$$

3. Since we have

$c_0 = 18+2 = 20$ $\quad c_1 = -3-4 = -7$ $\quad c_2 = 1$

$\gamma_0 = 12+2 = 14$ $\quad \gamma_1 = 1$ $\quad \gamma_2 = -2-3 = -5$

it follows that

$$P_1^* = \tfrac{14+100}{35-1} = \tfrac{57}{17} = 3\tfrac{6}{17} \quad \text{and} \quad P_2^* = \tfrac{20+98}{35-1} = \tfrac{59}{17} = 3\tfrac{8}{17}$$

Substitution into the given demand or supply function yields

$$Q_1^* = \tfrac{194}{17} = 11\tfrac{7}{17} \quad \text{and} \quad Q_2^* = \tfrac{143}{17} = 8\tfrac{7}{17}$$

Exercise 3.5

1.

(a) Three variables are endogenous: Y, C, and T.

(b) By substituting the third equation into the second and then the second into the first, we obtain

$$Y = a - bd + b(1-t)Y + I_0 + G_0$$

or

$$[1-b(1-t)]Y = a - bd + I_0 + G_0$$

Thus

$$Y^* = \frac{a-bd+I_0+G_0}{1-b(1-t)}$$

Then it follows that the equilibrium values of the other two endogenous variables are

$$T^* = d + tY^* = \frac{d(1-b)+t(a+I_0+G_0)}{1-b(1-t)}$$

and

$$C^* = Y^* - I_0 - G_0 = \frac{a-bd+b(1-t)(I_0+G_0)}{1-b(1-t)}$$

2.

(a) The endogenous variables are Y, C, and G.

(b) $g = G/Y$ = proportion of national income spent as government expenditure.

(c) Substituting the last two equations into the first, we get

$$Y = a + b(Y - T_0) + I_0 + gY$$

Thus

$$Y^* = \frac{a - bT_0 + I_0}{1 - b - g}$$

(d) The restriction $b + g \neq 1$ is needed to avoid division by zero.

3. Upon substitution, the first equation can be reduced to the form

$$Y - 6Y^{1/2} - 55 = 0$$

or

$$w^2 - 6w - 55 = 0 \qquad (\text{where } w = Y^{1/2})$$

The latter is a quadratic equation, with roots

$$w_1^*,\ w_2^* = \left[\frac{1}{2}6 \pm (36 + 220)^{1/2}\right] = 11,\ -5$$

From the first root, we can get

$$Y^* = w_1^{*2} = 121 \quad \text{and} \quad C^* = 25 + 6(11) = 91$$

On the other hand, the second root is inadmissible because it leads to a negative value for C:

$$C^* = 25 + 6(-5) = -5$$